# Oil Compressibility and Polytropic Air Compression Analysis for Oleopneumatic Shock Struts

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**The author has investigated various aspects of dynamic design of oleopneumatic landing gear shock absorbers. A review of available literature shows that effects on shock absorption characteristics due to hydraulic fluid compressibility, gas solubility and its entrainment-in-oil, and the nature of polytropic exponent during the compression stroke are often neglected by the designer. An attempt has been made to establish the importance of these parameters and possible means of representing them as mathematical models. The suggested models are capable of being incorporated into any realistic shock strut dynamics similation.**

## Nomenclature

$Aa$ = area of shock strut cylinder subjected to air pressure
$Cv$ = specific heat at constant volume
$J$ = heat equivalent of work (Joule's constant)
$k$ = specific heat ratio, $Cp/Cv$
$k'$ = thermal conductivity of strut cylinder
$n$ = polytropic exponent for air compression/expansion
$p$ = pressure
$Q$ = quantity of heat transferred
$R$ = gas constant
$S$ = shock strut stroke
$T$ = temperature absolute
$U$ = total internal energy
$V$ = volume
$w$ = weight of gas
$W$ = work done in the process
$\beta$ = oil bulk modulus

*Subscripts*

$o$ = oil
$a$ = air
$i$ = instananeous value of the variable
$0$ = strut extended or initial value of the variable

## Fluid Compressibility and Entrained Air

### Literature Review

THE value of the bulk modulus of a fluid in a hydraulic system used in design calculations is arbitrary, often modified from design to design, and varying greatly with the experience of the designer. The values of compressibility given in tables, or calculated from the various formulas given in standard references, are not necessarily the values which are effective in a particular application.[1] These reported values make no allowance for the effect of the stretching of the containing walls, and also neglect the effect of entrained air. Almost every hydraulic system draws in some air and mixes it thoroughly with the oil. The oil becomes saturated with the dissolved air and may also carry an appreciable volume of bubbles. This effect is greater if the liquid has a tendency to foam. The air that is in true solution apparently has little effect in most cases, but air bubbles may cause trouble. If the entrainment problem is serious, it may be necessary to provide for deaeration of the oil.[2]

At low pressures even a small bubble of air will greatly increase the effective compressibility of a considerable volume of oil. As the pressure increases, much of the air will dissolve in the oil, and since the compressibility of a gas varies inversely as the pressure, the effect of bubbles is much less at the higher operating pressures. Inasmuch as the effects of air bubbles and stretch of the walls are somewhat indeterminate, it is common practice to assume a constant effective bulk modulus.

Figure 1 shows the effects of entrained air; in the lower chamber of a shock strut cylinder, upon the load stroke curves of a modern jet landing gear. The data shown are experimental. To each curve there corresponds a different percentage of air entrapped in the lower chamber. It is seen that small amounts of air may alter the shape of the load stroke curves considerably, reducing the gear load in the initial part of the stroke and increasing it towards the compressed position.

Magorien[2] experimentally determined the effects of entrained and/or dissolved air in a hydraulic fluid flowing through an orifice/actuator mechanism. He reported that for a content of only 0.17% (by volume) of compressible entrained air at 3000 psig, the theoretical bulk modulus is cut in half. The results of the high pressure cycling indicate (see Fig. 2) that systems using air-saturated MIL-H-5606 fluid (or similar hydraulic fluids) at pressures of approximately 1000 psig or greater, can look forward to the generation of entrained air across orifices in the system.

Due to an excessive number of airline reports pertaining to landing gear shock strut servicing difficulties, a Boeing investigation revealed the following.

The combined effects of oil compressibility, strut expansion, servicing procedures that allow trapped air pockets to remain within the strut, and the formation of an oil-gas chemical solution make the theoretical load stroke curve impractical. Laboratory tests on a typical shock strut verified the theory that oil properties such as bulk modulus and volume are affected very little by the presence of a dissolved gas. Petroleum oils in general will dissolve $8.5 \pm 0.5\%$ by volume of air at atmospheric pressure and room temperature. For pressures higher than atmospheric, absorption follows Henry's law. A practical upper limit exists for this law where the oil becomes saturated with the gas, and as such, increased pressures result in very little gas entering into solution with the oil. With MIL-H-5606 oil and nitrogen gas, this was found to occur between 190 and 250 psig.

When the gas does enter the oil in solution, the net effect to a closed pressurized gas-oil system such as a shock strut is a ratio shift of the *effective* gas-oil relationship between the time the ratio is originally established with new oil, and the time when the oil finally becomes saturated with the gas.

The effective bulk modulii of three different shock strut systems using the same hydraulic fluid were determined to be 113,000 psi-220,000 psi.

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## Polytropic Air Compression in Shock Strut

In literature dealing with the design of landing gears, some investigators, in calculating the pneumatic spring force, have assumed the air compression process to be virtually adiabatic. Some have assumed the process to be isothermal and still others have considered some intermediate polytropic process. Lack of experimental data on the air compression process in shock struts is the main reason for such a divergence of opinion.

Although many drop tests have been made, not many include the direct measurement of air pressures throughout the compression and expansion strokes. Hurty,[3] Walls,[4,6] and Milwitzky, et al.[5] analyzed the measurements of air pressure and strut stroke obtained in various tests on several small landing gears (drop wt = 1500-2500 lbs) and concluded that the effective polytropic exponent may be in the neighborhood of 1.1 for practical cases. Milwitzky also pointed out that the effective polytropic exponent "$n$" depends upon the rate of compression and the rate of heat transfer from the air to the surrounding environment. The actual thermodynamic process is complicated by the violent mixing of the highly turbulent efflux of hydraulic fluid and the air in the upper chamber during impact.

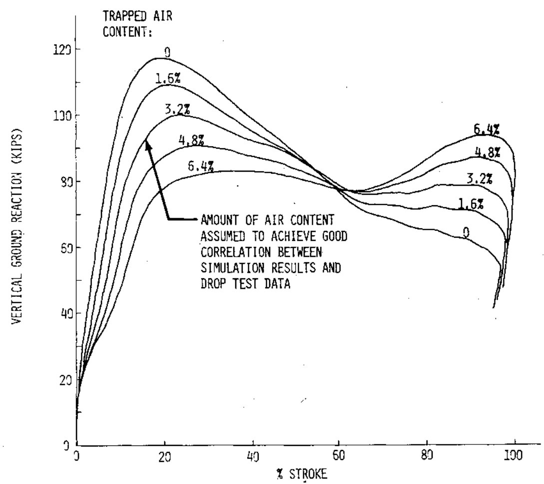


**Fig. 1 Effects of entrained air.**

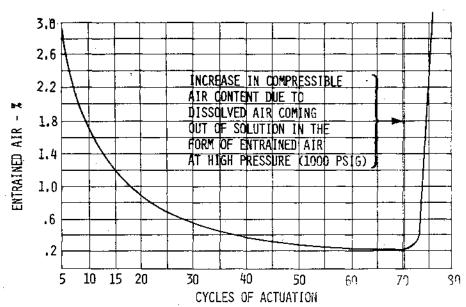


**Fig. 2 Air content generated from dissolved air.**

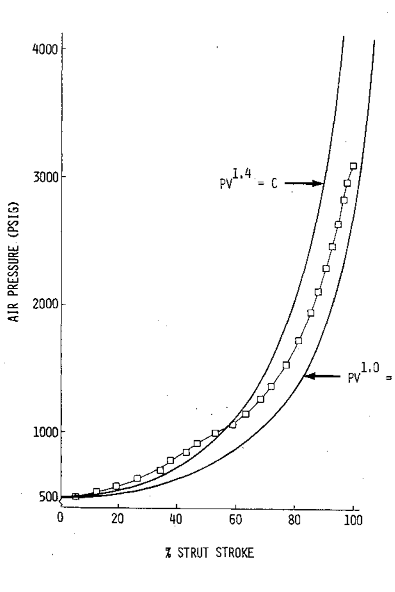


**Fig. 3 Nose gear shock strut comparison between drop test air pressure record and theoretical values.**

Fig. 3 shows a typical air pressure characteristic from the drop test data of a modern jet nose landing gear. It can be seen that the air compression process approximates a polytropic process with an exponent of 1.2.

For the main landing gear of the same airplane, a curve depicting the oleo pressure vs oleo stroke relationship is shown in Fig. 4. The calculations of the polytropic exponent show the range for $n$ to be from 0.87-1.015. It is therefore clear that no consensus exists as to the exact value of the polytropic exponent. The compression stroke probably begins with a polytropic exponent close to the adiabatic constant for air decreasing in value as the compression stroke progresses, and then dropping still further at the end of the stroke and during the expansion stroke. The loop thus formed probably represents heat loss through the walls of the strut during the process.

## Mathematical Model for Hydraulic Fluid Compressibility

The equation of state for air and the hydraulic fluid are, respectively

air: $$v_a = v_{a_0}(pa_o/pa)^{1/n} \tag{1}$$

oil: $$v_o = v_{o_0}[1-(P_a - Pa_o)/\beta] \tag{2}$$

The sub-subscript "$0$" refers to the strut extended or initial value. The instantaneous air-oil volume of the shock strut may be written as a function of the stroke, $S$, as follows

$$v_o + v_a = v_{o_0} + v_{a_0} - A_a(S)$$

$$v_a = v_{o_0} - v_o + v_{a_0} - Aa(S)$$

Substituting for $v_o$ from (2) we get

$$v_a = v_{o_0}\frac{(Pa - P_{a_0})}{\beta} + v_{a_0} - Aa(S) \tag{3}$$

also

$$Pa = Pa_o\left(\frac{v_{a_0}}{v_a}\right)^n \tag{4}$$

Substituting for $v_a$ from (3) results in

$$Pa = Pa_o\left[\frac{v_{a_0}}{v_{o_0}\dfrac{(pa - pa_o)}{\beta} + v_{a_0} - Aa(S)}\right]^n$$

or

$$\frac{P_a}{P_{a_0}} = \left[\frac{1}{\dfrac{P_{a_0}v_{o_0}}{\beta v_{a_0}}\left(\dfrac{Pa}{Pao} - 1\right) + 1 - \dfrac{A_a}{v_{a_0}}(S)}\right]^n \tag{5}$$

If we now define the following quantities

$$\frac{Pa}{P_{a_0}} = P. \tag{5a}$$

$$\frac{P_{a_0} v_{o_0}}{\beta v_{a_0}} = A \tag{5b}$$

$$\frac{A_a}{v_{o_0}} = B \tag{5c}$$

then Eq. (5) may be written as

$$(P) = \left[\frac{1}{A(P-1)+1-B(S)}\right]^n$$

Differentiating with respect to time, we may write

$$(\dot{P}) = {}^{-n}\frac{P}{[A(P-1)+)-B(S)]}[A(\dot{P})-B(\dot{S})]$$

or

$$(\dot{P}) = \frac{nBP(\dot{S})}{[(P)A(n+1)-A+1-B(S)]} \tag{6}$$

Fig. 5 is an analog block diagram representation of Eqs. (5a), (5b), (5c), and (6).

## Heat Transfer Analysis for Exponent "$n$"

### Approach I

Thermodynamically, the compression/expansion of a gas within a shock strut is a nonflow process. In order to subject a quantity of fluid to a nonflow process, two quantities must be transferred across the boundary of the closed system, viz. an amount of heat, $Q$, and mechanical work, $W$. For a reversible nonflow ideal gas process, assuming a constant specific heat, it can be shown that

$$_1Q_2 = \frac{W}{J}\frac{k-n}{k-1} \tag{7}$$

The heat exchange has been expressed as a proportion of the work done, thus, for a given amount of positive work (expansion), the heat transferred to the gas decreases with an increase of $n$ until, when $n$ equals $k$, it is zero. Further increase in the value of $n$ gives a negative value of heat transfer, i.e., heat is rejected by the gas.[7] Thus, the value of the index $n$ depends on the ratio of the heat transfer to the work done. The heat transferred to the surroundings during compression determines the value of $n$. As the work done is negative and the value of $n$ lies between 1 and 1.4, the heat transferred will always be negative. Again using applicable equations for a reversible nonflow process, it can be shown that

$$Q = -\frac{k-n}{n-1}C_v w T_1\left[\left(\frac{p_2}{p_1}\right)^{(n-1)/n} - 1\right] \tag{8}$$

Equation (8) thus represents the variation of "$n$" in terms of the transferred heat.

### Approach II

Using basic gas laws, the exponent "$n$" can be expressed in linear terms as follows: $pv^n = \text{constant}$, and $pv = wRT$ or $p = wRT/v$. Combining these relationships results in: $Tv^{n-1} = C$. Differentiating with respect to time gives

$$(n-1) = -\frac{\dot{T}v}{T\dot{v}}$$

or

$$n_i = (1 - \frac{\dot{T}_i V_i}{T_i \dot{V}_i}) \tag{9}$$

The instantaneous volume $v_i$ can be expressed as

$$v_i = v_o - A_a S \tag{10}$$

Differentiating (10) with respect to time

$$\dot{v}_i = -A_a \dot{S} \tag{11}$$

Now all that remains is to obtain an expression for $T_i$. From 1st Law

$$Q = U_2 - U_1 + \frac{W}{J} \tag{12a}$$

Also

$$W = \int_1^2 p dv \tag{12b}$$

$$U_2 - U_1 = wc_v(T_2 - T_1) \tag{12c}$$

Differentiating Eqs. (12a), (12c), integrating Eq. (12b) and mutually substituting gives

$$\dot{Q} = wC_v(\dot{T}_i) + \frac{1}{J}(p_i)(\dot{v}_i) \tag{13}$$

where the subscript 2 has been replaced by the subscript $i$. Neglecting heat loss through radiation, the rate of heat transfer across a cylindrical surface of radius $r$ and length $S$ can be written as

$$\dot{Q} = \frac{dQ}{dt} = -k' 2\pi rs\frac{dT}{dr} \tag{14}$$

where $k'$ = the thermal conductivity and $dT/dr$ is the temperature gradient at the radius $r$.

The previous equation assumes a homogeneous tube wall and the temperature is assumed to vary only with radial distance from the axis, i.e., the temperature gradient is wholly in the radial direction so that

$$\dot{Q}\int_{r_i}^{ro}\frac{dr}{r} = -2\pi k' S\int_{T_i}^{T_o} dT$$

or

$$\dot{Q} = 2\pi k' S(T_i - T_o)/\ln(r_o/r_i) \tag{15}$$

Equating the right-hand-side of Eqs. (13) and (15) and rearranging gives

$$(\dot{T}_i) = \left[\frac{-2\pi k'}{wC_v\ln(r_o/r_i)}\right](S)T_i - T_o - \frac{1}{JwC_v}(p_i\dot{v}_i) \tag{16}$$

Also

$$p_i = wR\frac{T_i}{v_i} \tag{17}$$

Combining Eqs. (10), (11), (16), and (17) we get

$$(\dot{T}_i) = -\left(\frac{2\pi k'}{wC_v \ln r_o/r_i}\right)(S)(T_i - T_o) + \frac{Aa(k-1)}{v_o}\left\{\frac{T_i}{1-[A_a(S)/v_o]}\right\}(\dot{S}) \tag{18}$$

where $k = C_p/c_v$. Equations (9-11) and (18) are directly usable for analog simulation and represent the variation of the polytropic exponent "$n$" for both compression and extension strokes. The derivation of Eq. (18) does not account

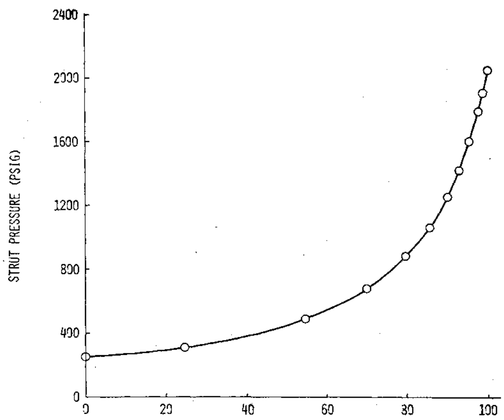


**Fig. 4 Experimental static pressure-stroke curve.**

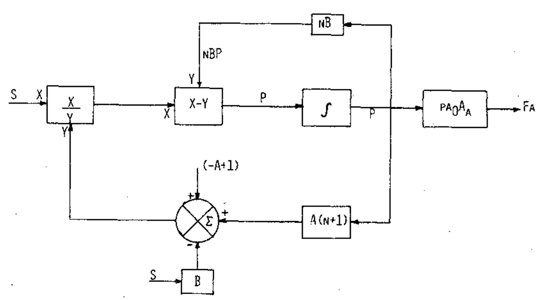


**Fig. 5 Fluid compressibility model.**

for the oil cooling effect, and any further improvements in this regard will necessitate considerable controlled testing.

The heat transfer analysis just presented was based on a steady state heat exchange between the system and the surroundings. The actual process, however, due to dynamic situation, is one under transient conditions. It is this author's belief, nevertheless, that it is a step forward in the right direction to treat the polytropic exponent "$n$" as a variable and not as a constant.

## Mathematical Model for Variable "$n$"

$$w = \frac{p_o v_o}{RT_o} \tag{19}$$

Combining Eqs. (9), (11), and (19) gives

$$(n_i) = 1 + [p_o v_o (\dot{T}_i) / T_o A_a (\dot{S}) (P_i)] \tag{20}$$

Combining Eqs. (18) and (19)

$$(\dot{T}_i) = \frac{-2T_o \pi k' R}{p_o v_o c_v \ln(r_o/r_i)} (S)(T_i - T_o) + \frac{A_a(k-1)}{v_o} \left\{ \frac{T_i}{1-[Aa(S)/vo]} \right\} (\dot{S}) \tag{21}$$

Combining Eqs. (10), (17), and (19) gives

$$(p_i) = \frac{p_o}{T_o} \left\{ \frac{T_i}{1-[Aa(S)/vo]} \right\} \tag{22}$$

Figure 6 is an analog block diagram representation of Eqs. (19)-(22).

## Conclusions and Recommendations

1) Laboratory tests seem to verify the theory that hydraulic fluid properties such as bulk modulus and volume are affected very little by the presence of a dissolved gas. Entrained air, however, even in very small percentages, affects the load stroke characteristics considerably. Compressibility effects of the fluid must, therefore, be taken into account for analysis, simulation, or when establishing servicing procedures, for shock strut. Based on the results of Fig. 1, it is suggested that 3 to 5% of air by volume be assumed as entrained in oil for preliminary analysis.

2a) Effective bulk modulus for any specific system is different from that of any other system using identical fluid. Actual measurements must, therefore, be made to establish this variable rather than relying upon "text book" values.

2b) The change in strut stroke due to oil compressibility could be represented as $V_{oil}/A\beta\ (P_l - P_2)$ where $P_l$ and $P_2$ are instantaneous and extended position pressures. Similarly, the change in stroke due to strut deformation within the elastic range could be expressed as $K_e(P_l - P_2)$ where $K_e$ is the strut elastic expansion constant. Thus, the combined effect can be used to approximate the total deviation from state equation $PV = mRT$. The total deviation of the load stroke curve from the ideal curve can be determined experimentally. An empirical value of $n$ can be determined from this data. This, in turn, would establish the bulk modulus for a given system.

3) The polytropic exponent "$n$" is not a constant. From the energy standpoint, "$n$" could be assumed to remain constant but not isothermal. For a rigorous study of the system, however, the heat transfer characteristics must be considered, e.g., the present study.

4) Additional experimental data is needed to further clarify the type of air compression process actually obtained. Measurement of strut pressures under dynamic conditions for future drop test and flight test programs is strongly recommended.

5) Experimental data evaluating effects of parameters like drop weight, type of air chamber, and cooling effect of oil on the air compression process (exponent), must be obtained to quantify these effects.

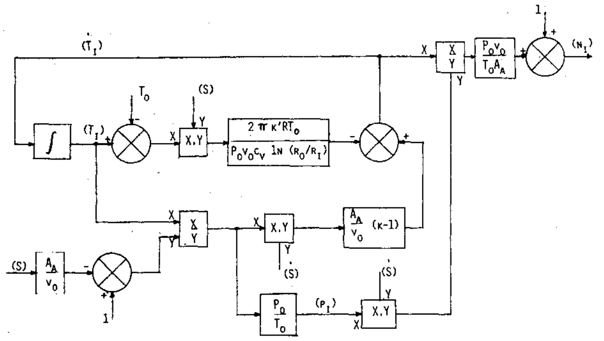


**Fig. 6 Polytropic air compression model.**

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